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ASPECTS ABOUT THE VIBRATIONS OF A LINEAR VISCOELASTIC CINEMATIC ELEMENT OF CRANK AND CONNECTING ROD ASSEMBLY

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Abstract: This work presents the vibrational strains for a viscoelastic cinematic element of crank and connecting rod assembly. The movement equations of the linear-elastic straight kinematic elements in plan-parallel motion are presented by using the Hamilton's variational principle. In order to obtain the dynamic response, the Laplace integral transforms and the finite Fourier transforms are applied. Finally, for a concrete case, the plotting of the displacements are given.

1. THE ANALYTIC DETERMINATION OF THE DISPLACEMENTS FIELD FOR A VISCOELASTIC CINEMATIC ELEMENT OF A CRANK AND CONNECTING ROD ASSEMBLY

By applying in the movement's mathematical model of a linear elastic cinematic element bar type [1], the Laplace one-sided transform proportional to time and replacing the E modulus with $\tilde{E}(s)$ the matriceal equation of the first aproximation in Laplace images is obtained for the vibrations of the viscoelastic connecting rod of the R(RRT) mechanism figured in Fig. 1, as follows:

$$[L_0(s)]\{\tilde{u}^{(1)}\} + [M_4]\{\tilde{a}_0\} + \{\tilde{V}_1\} = \{0\} \tag{1}$$

where:

$$[L_0(s)] = [M_1(s)]\frac{\partial \bullet}{\partial x^4} + [M_8(s)]\frac{\partial \bullet}{\partial x^2} + [M_4(s)]\bullet; [M_1(s)] = \tilde{E}(s)\cdot I \cdot \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[M_8(s)] = \begin{bmatrix} \tilde{E}(s)A & 0 \\ 0 & -\rho \cdot I \cdot s^2 \end{bmatrix}; [M_4(s)] = s^2[M_4]; \{\tilde{a}_0\} = \{\tilde{a}_{01}(s); \tilde{a}_{02}(s)\}^T;$$

$$(\tilde{V}_1) = \rho A x \{\tilde{g}(s); \tilde{\varepsilon}(s)\}^T; \omega^2(t) = g(t); \{\tilde{u}^{(1)}\} = \{\tilde{u}_1^{(1)}(x,s); \tilde{u}_2^{(1)}(x,s)\}^T;$$

$$\tilde{E}(s) = \frac{a_0 s}{b_0 s + b_1}; a_0 = 9GK; b_0 = 3K + G; b_1 = \frac{3KG}{\eta};$$

$$K = \frac{\nu E}{(1+\nu)(1-2\nu)} + \frac{2}{3}G;$$

$\tilde{E}(s)$ - is the the mathematical expression of Maxwell mechanical model for solid state polymers;
G – Transversal elasticity modulus;
K – compresibility modulus;
υ - Poisson's transversal elasticity coefficient;
η - Maxwell's model newtonian component constant.

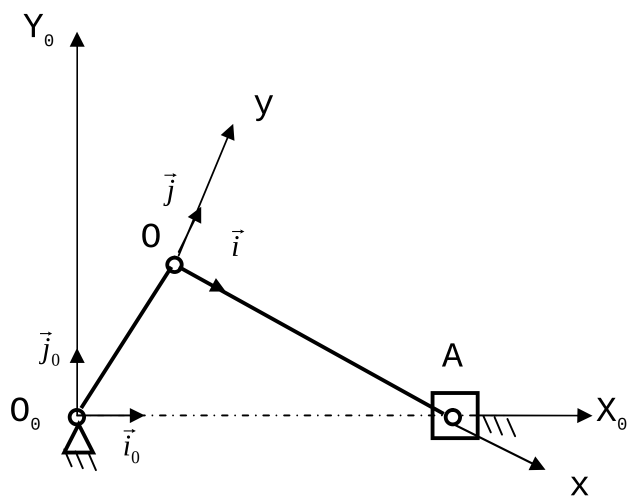

Fig. 1 The crank and connecting rod mechanism

Applying in (1) the finite Fourier transforms in cosine and sine, to the first and respectively to the second equation, there are obtained uncoupled algebraic systems having as unknowns the displacements $u_{1,c}^{(1)}(n,s)$ and $u_{2,s}^{(1)}(n,s)$ in Laplace and Fourier images in cosine, respectively in sine. Then, by inverting the Laplace and Fourier transforms, there results the solution in the first approximation $\{u^{(1)}(x,t)\}$. With $\{u^{(1)}(x,t)\}$ it is calculated, in a first approximation, the vector $\{F^{(1)}\}$ which is introduced into the equation (1'), resulting the mathematical model in the second approximation, whose resolution can be obtained with the help of the integr al transforms, finally having the solution $\{u^{(2)}(x,t)\}$ in the second approximation. Continuing the iterative process, results the mathematical model in the approximation ,,j".

$$[L_0]\{u^{(j)}(x,t)\}+[M_4]\{a_0\}+\{V\}_1+\{F^{(j-1)}\}=\{0\} \tag{2}$$

where:

$$\{F^{(j-1)}\}=[M_6]\{u^{(j-1)}(x,t)\},\ j=1,2,...,n;\ \{F^{(0)}\}=\{0\}\ ;$$

$$\{u^{(j)}\}=\{u_1^{(j)}(x,t),u_2^{(j)}(x,t)\}^T\ ;$$

$$\{u^{(j-1)}\}=\{u_1^{(j-1)}(x,t),u_2^{(j-1)}(x,t)\}^T$$

The solution in ,,j" aproximation will be:

$$u_1^{(j)}(x,t)=\frac{1}{L}\cdot u_{1,c}^{(j)}(o,t)+\frac{2}{L}\sum_{n=1}^{n=\infty}u_{1,c}^{(j)}(n,t)\cdot\cos(\alpha_n\cdot x) \tag{3}$$

$$u_2^{(j)}(x,t)=\frac{2}{L}\sum_{n=1}^{n=\infty}u_{2,s}^{(j)}(n,t)\cdot\sin(\alpha_n\cdot x), \tag{4}$$

Where:

$u_{1,c}^{(j)}(n,t)$ şi $u_{2,s}^{(j)}(n,t)$ are the finite Fourier transforms in cosine, respectively in sin us of the elastic longitudinal and transversal displacement.

The process of the successive approximation is considered complete when $\forall n, \left\|\{u^{(j)}\}-\{u^{(j-1)}\}\right\|\le\varepsilon,$, where:

$\varepsilon>0$ and low enough depending on the precision of the operation required;

And:

$$\left\{u^{(j)}\right\} = \left\{u_1^{(j)}(x,t), u_2^{(j)}(x,t)\right\}^T; \tag{5}$$

$$\left\{u^{(j-1)}\right\} = \left\{u_1^{(j-1)}(x,t), u_2^{(j-1)}(x,t)\right\}^T. \tag{6}$$

The connecting rod OA being double-articulated, the just on the line conditions which allowed the application of the two Fourier transforms, for initial functions and for their Laplace images, were:

$$\frac{\partial u_1(0,t)}{\partial x} = \frac{\partial u_1(L,t)}{\partial x} = 0; u_2(0,t) = u_2(L,t) = 0; \frac{\partial^2 u_2(0,t)}{\partial x^2} = \frac{\partial^2 u_2(L,t)}{\partial x^2} = 0,$$
$$\frac{\partial u_1(0,s)}{\partial x} = \frac{\partial u_1(L,s)}{\partial x} = 0; u_2(0,s) = u_2(L,s) = 0; \frac{\partial^2 u_2(0,s)}{\partial x^2} = \frac{\partial^2 u_2(L,s)}{\partial x^2} = 0. \tag{7}$$

The initial conditions have been admitted:

$$u_1(x,0) = 0; \frac{\partial u_1(x,0)}{\partial t} = 0; u_2(x,0) = 0; \frac{\partial u_2(x,0)}{\partial t} = 0.$$

The transversal displacements fields, in the first approximation, in the case of free vibrations of the connecting rod OA belonging to the mechanism R(RRT) in figure 1, represented by the equation (3), where:

$$\{a_o\} = \left\{\omega_0^2 r\left[\frac{r}{L}\sin^2(\omega_0 t) - \cos(\omega_0 t)\right]; -\omega_0^2 r\left[\sin(\omega_0 t) + \frac{r}{2L}\sin(2\omega_0 t)\right]\right\}^T;$$
$$\omega = -\frac{r}{L}\omega_0\cos(\omega_0 t); \varepsilon = \frac{\omega_0^2 r}{L}\sin(\omega_0 t), \tag{8}$$

will be supplied, as a result of the application of the preceding algorithm of integration, by the function:

$$u_2^{(1)}(x,t) = \frac{2}{L}\sum_{n=1}^{n=\infty} u_{2,s}^{(1)}(n,t)\cdot\sin(\alpha_n \cdot x), \tag{9}$$

where:

$$u_{2,s}^{(1)}(n,t) = \frac{a_{n,4}}{a_{n,1}}\{a_{n,3}\omega_0[\cosh(\Omega_{n,1}t) - \sinh(\Omega_{n,1}t)][a_1 + a_{n,5} -$$
$$2a_2 a_{n,1}\sinh(\Omega_{n,1}t) - 2a_2 a_{n,1}\cosh(\Omega_{n,1}t) +$$
$$+ a_2 a_{n,1} + a_{n,6}\sinh(\omega_n t) + a_{n,6}\cosh(\omega_n t)]\} + \frac{a_{n,7}\omega_0}{a_{n,1}a_{n,2}}$$
$$\left[\cosh\left(\frac{b_1}{b_0}t\right) - \sinh\left(\frac{b_1}{b_0}t\right)\right]\{a_{n,8}[-1 + \cosh(\omega_n t) + \sinh(\omega_n t)]$$
$$[\cosh(\Omega_{n,2}t) + \sinh(\Omega_{n,2}t)] + a_3 a_{n,1}\omega_0\sin(\omega_0 t)$$
$$\left[\cosh\left(\frac{b_1}{b_0}t\right) + \sinh\left(\frac{b_1}{b_0}t\right)\right] - a_{n,9}\cdot[\cosh(\Omega_{n,2}t) + \sinh(\Omega_{n,2}t)][-b_1 a_2 +$$
$$+ b_1 a_{n,1} + a_{n,10}\cosh(\omega_n t) + a_{n,10}\sinh(\omega_n t) + a_4 + 2(-b_1 a_{n,1}\cdot$$
$$\cdot\cos\omega_0 t + b_0 a_{n,1}\omega_0\sin\omega_0 t)\cosh(\Omega_{n,1}t) + 2a_{n,1}(-b_1\cos\omega_0 t + b_0\omega_0\sin\omega_0 t)\cdot$$
$$\cdot\sinh(\Omega_{n,1}t)]\} + \frac{a_{n,11}\omega_0}{a_{n,1}a_{n,2}}\cdot\left[\cosh\left(\frac{b_1}{b_0}t\right) - \sinh\left(\frac{b_1}{b_0}t\right)\right]$$

$$\begin{aligned}&\{\left(a_{n,8}-a_{n,9}a_2b_1+4b_0^2\omega_0^2a_{n,9}a_6\right)\\&[-1+\cosh(\omega_n t)+\sinh(\omega_n t)]\cdot[\cosh(\Omega_{n,2}t)+\sinh(\Omega_{n,2}t)]+\\&a_{n,1}\left[a_5\omega_0\sin(2\omega_0 t)-2a_{n,9}b_1\cos(2\omega_0 t)+4b_0^2\omega_0^2a_{n,9}\sin(2\omega_0 t)\right]\cdot\\&\cdot\left[\cosh\left(\frac{b_1}{b_0}t\right)+\sinh\left(\frac{b_1}{b_0}t\right)\right]+a_{n,9}b_1a_{n,1}[\cosh(\Omega_{n,2}t)+\sinh(\Omega_{n,2}t)]\\&+a_{n,9}b_1a_{n,1}[\cosh(\Omega_{n,1}t)+\sinh(\Omega_{n,1}t)]]\}\end{aligned} \tag{10}$$

and:

$$\begin{aligned}&a_1=-A\rho b_1^2;\ a_2=b_1\sqrt{A\rho};\ a_3=2A\rho\sqrt{A\rho}\left(b_1^2+b_0^2\omega_0^2\right);\ a_4=-2\sqrt{A\rho}\cdot b_0^2\omega_0^2;\\&a_5=4A\rho\sqrt{A\rho}\left(b_1^2+4b_0^2\omega_0^2\right);a_6=2\sqrt{A\rho};a_{n,1}=\sqrt{A\rho b_1^2-4\cdot I\cdot a_0b_0\alpha_n^4};\\&a_{n,2}=A^2\rho^2\omega_0^2\left(b_1^2+b_0^2\omega_0^2\right)+I\cdot a_0\alpha_n^4\left(I\cdot a_0\alpha_n^4-2A\rho b_0\omega_0^2\right);\\&a_{n,3}==-4\sin(L\alpha_n)-r\cdot\alpha_n\cos(L\alpha_n)+\alpha_n(r+4L);a_{n,4}=-\frac{r\cdot\sqrt{A\rho}}{8L\cdot I\cdot a_0\alpha_n^6};\\&a_{n,5}=2\cdot I\cdot a_0b_0\alpha_n^4;a_{n,6}=a_2a_{n,1}+a_2^2-a_{n,5};\\&a_{n,7}=\frac{r\sqrt{A\rho}\left[\sin(L\alpha_n)-L\alpha_n\right]}{2L\alpha_n^2};\\&a_{n,8}=2\cdot I^2\cdot a_0^2b_0\alpha_n^8;\ a_{n,9}=I\cdot\sqrt{A\rho}\cdot a_0\alpha_n^4;\\&a_{n,10}=a_2b_1+b_1a_{n,1}-a_4;a_{n,11}=\frac{r^2\sqrt{A\rho}\left[\cos(L\alpha_n)-1\right]}{8L\alpha_n};\end{aligned} \tag{11}$$

$$\begin{aligned}&\Omega_{n,1}=\frac{b_1}{2b_0}+\frac{\omega_n}{2};\\&\Omega_{n,1}=\frac{b_1}{2b_0}+\frac{\omega_n}{2};\Omega_{n,2}=\frac{b_1}{2b_0}-\frac{\omega_n}{2};\\&\omega_n=\frac{a_{n,1}}{b_0\cdot\sqrt{A\rho}}.\end{aligned} \tag{12}$$

2.NUMERICAL APPLICATION FOR A REAL RRT MECHANISM

It is considered a numerical application of RRT mechanism as follows :
L=1 $[\mathrm{m}]$; r=0,07 $[\mathrm{m}]$; ω_0 =31,41 $\left[s^{-1}\right]$; b=0,011 $[\mathrm{m}]$; h=0,011 $[\mathrm{m}]$; ρ=1213,3 $\left[Kg/m^3\right]$;
G=0,0118979 $\cdot 10^{11}\left[\frac{N}{m^2}\right]$; $\eta=4{,}5\cdot 3600\cdot 10^{13}\left[\frac{Ns}{m^2}\right]$; K=0,02424455 $\cdot 10^{11}\left[\frac{N}{m^2}\right]$.

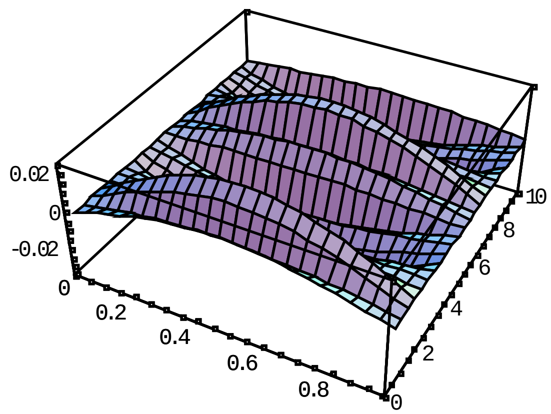

Fig.2 The transversal displacement $u_2 = u_2^{(1)}(x,t)$

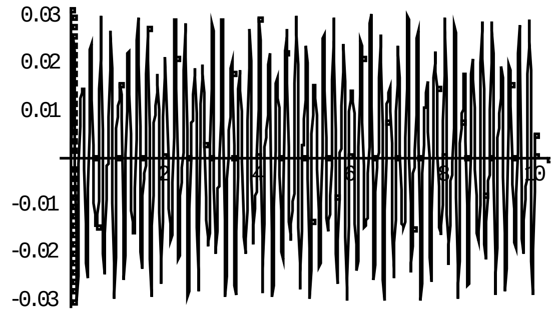

Fig.3 The transversal displacement variation at x=L/2: $u_2 = u_2^{(1)}\left(\frac{L}{2}, t\right)$

Because the longitudinal displacement field has a negligible influence [1], it has continued the iterative/repeated process only for transversal displacements.
The mathematical model (8) shows the influence of the cinematic parameters upon the transversal displacements and in the same time upon the stress and strain fields. These influences are very important in dimensioning and designing products.

3. CONCLUSIONS

This paper is useful is the machine designing. The transversal displacements field, this way obtained, constitutes the support of further determination of the stress and deformation state. The cinematic parameters of movement have an important influence concerning on the vibrations as the analytic expresion (8) indicate. The main purpose was in determination of the transversal displacements field because the influence of the longitudinal displacements towards the indirect stress or towards the state of deformation is quite minor.

REFERENCE

1. Băgnaru, D., Rinderu, P., Vibratiile sistemelor elastice, Editura LOTUS Craiova, 1998.
2. Băgnaru D. - Vibraţiile elementelor cinematice, Editura SITECH, Craiova, 2005, ISBN 973 -657-854-2.

3. Băgnaru D., Marghitu D.B. – Linear vibrations of viscoelastic links, 20-th Southeastern Conference on Theoretical and Applied Mechanics, April 16 – 18, 2000, Callaway Gardens and Resort, Pine Mountain, Georgia, USA, pag. 1 – 7.
4. Băgnaru D., Cătăneanu A. – Calculus methods in dynamics of cinematic chains with linear viscoelastic elements, International Conference Beyond 2000, Engineering Research Strategies, 25 – 26 nov. 1999, Sibiu, pag. 225 – 228.
5. Bolcu D., Băgnaru D. – Aplicaţii ale transformărilor integrale în mechanică, Editura Universitaria, Craiova, 2004, ISBN 973-8043-568-X.
6. Ditkine, V., Proudnikov, A., Transformation integrales et calcul operationnel, Ed. Mir, Moscou, 1978.
7. Nowacki, W., Dinamica sistemelor elastice, Editura Tehn ica, Bucuresti, 1969.